APPLICATION OF FRACTAL DIMENSION AND ENTROPY AS THE MEASURES FOR RECOGNIZING OFFSHORE VEGETATION ON AERIAL PHOTOGRAPHS

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ABSTRACT

One of the certificates that characterizes ecological landscape changeability is fractal dimension. According to Environmental Protection Agency, the dynamics and vectors of those changes can be determined in three-dimensional space where fractal dimension makes one of the axes. Entropy can be regarded as another measurement of changeability (chaos). Application of such measurement of landscape changeability as comparative patterns of land cover is the subject matter of the paper.

A fragment of a vertical color photograph of lake Lukajno shoreland taken by the author of the paper in September 2006 has been chosen for research.

The concept of fractal dimension does not have only one definition. Hence, so numerous methods of its calculating. Proposed by Clarke method of triangular truncated prism has been applied in research. The surface of the photograph was divided into 25 by 25 pixel squares. For each of them, in channels R, G, B fractal dimension and entropy were calculated. The created mosaics represent the distribution of local fractal and entropy dimension values.

Three vegetation concentrations and lake bed deprived of vegetation were chosen for land cover analysis. The chosen vegetation concentrations include osiers, elodeids (vegetation immersed in water) and heliophites (rushes). Standard values of dimension and entropy were determined for each of them as taken from 90 representative fields. In channels R, G, B mean values were calculated and variance-covariance matrix were defined. Applying Baran's method, elements of pattern mean error ellipse were calculated for each category of cover respectively.

The classification of the pixels of local value of dimension and entropy was based on the assumption that pixels whose values R, G, B are to be found inside the ellipse of a given category, belong to the category of determined probability.

Two classifications were carried out, with assumed probability equal 50%, depicting them as colorful rasters. The visual estimation of the correctness of vegetation site recognition has confirmed the applicability of recognition features.

In order to increase the probability of the pixel affinity do ellipse of a given class to 75%, classification was carried out in which it is assumed that a pixel belongs to an ellipse of a given class in the spaces of both recognition features.

Visual analysis of the classification results has confirmed that both fractal dimension and entropy are essential recognition features of vegetal cover relating to water habitat.

Erroneous classification or lack of category recognition might be caused by heterogeneity of vegetation patches and high sensitivity of the method to technical imperfection of information carrier.

Vegetation mosaic configurations do not show any distinct interior limits and vegetation concentrations can not be treated as pure fractals but rather as a blend of such, namely multifractals.

Keywords: fractal dimension, entropy, air photography, Luknajno lake

INTRODUCTION

The subject of this analysis is a fragment of an aerial color photograph in natural colors, depicting broadly-understood transition zone between mainland and water basin. In that zone, four classes of land cover were chosen for research purposes: helophytes (rushes), osier foliage, elodeas (submergent plants), and basin floor without any vegetation. In the image, those areas do not have distinct visible borders between one another. The patches of selected vegetation merge alongside the blurred ecotones. The landscape of that zone possesses no geometrical qualities, characteristic for cultivated fields.

The traditional approach to the classification of land cover on the basis of aerial photography in which the values of singular pixels are taken into account, with no proper reference nor the analysis of their surroundings, might be misleading in that case. The vegetation of coastal area is a dynamic structure, with various processes ongoing chaotically. The analysis of the spatial system of textural diversity of an image lays at the very basis of the object-based classification of such systems.

In the field of remote sensing, the characteristics most typically applied for describing chaotic systems are variance and entropy. It is also becoming more common to include methods of fractal analysis in the description of texture. Despite the lack of a unanimous definition [1], fractal dimension has become the measure for disorderedness and dynamics of changes in ecosystems [2].

According to the Environmental Protection Agency, the dynamics and direction of those changes can be determined with time in 3D space, for whose axis is constituted by the following measures (metrics): diversity measure, dispersion measure and fractal dimension.

Fractal geometry is a procedure which enables for describing and classifying selected space, and estimating the fractal dimension of image structures is one of the established methods of dimensioning. It allows both for selecting homogeneous areas and forms, and their generalization for the purpose of mapping.

METHODS AND MATERIALS

The source-material of the analysis is a photographic image of the north-eastern part of Luknajno lake (Fig. 1). The color image was scanned with the EPSON PERFECTION V750 PRO scanner of 2500 DPI resolution in the color mode R,G,B 8 bits per channel. With the picture scale of 1:32000, the acquired terrain resolution was approx. 0.325 m.

Luknajno lake is an overgrown and eutrophic flattened reservoir (average depth is approx. 0.6 m), where littoral zone encompasses all area. In the aerial photograph, one can notice not only floating or submergent vegetation, but also underwater meadows of Chara sp. The strip of submergent vegetation consists of reed and sedge rushes, with patches of willow bushes, accompanied by alder swamps. Those structures appear to show self-similarities, at least to a certain degree of magnification.

The shores surrounding the basin are flat. The wide area of land around the lake is a place of active agriculture, with a mosaic of fields and fallow lands, identified by their different textures.

Fig. 1. A fragment of the photograph used for the research, a – natural colors, b – R channel

CALCULATING THE FRACTAL DIMENSION

The concept of fractal dimension (Df) was introduced by Mandelbrot [3]. However, it is not yet a unanimously defined concept, and what makes the matter even more complex, mathematicians propose a few different definitions for it. Thus, there are many methods for calculating fractal dimension. One of the more frequently used is the triangular prism method [4]–[7]. It is similar to measuring size with the compass method [1], [3]. The estimated size calculated with this method is ca. 1÷2.

The area of a roughened plane may be calculated with varying precision, depending on different measure values. With gradually increasing precision, larger surface areas are acquired. This dependency constitutes the basis for estimating fractal dimension. The details of that technique are illustrated in Fig. 2.

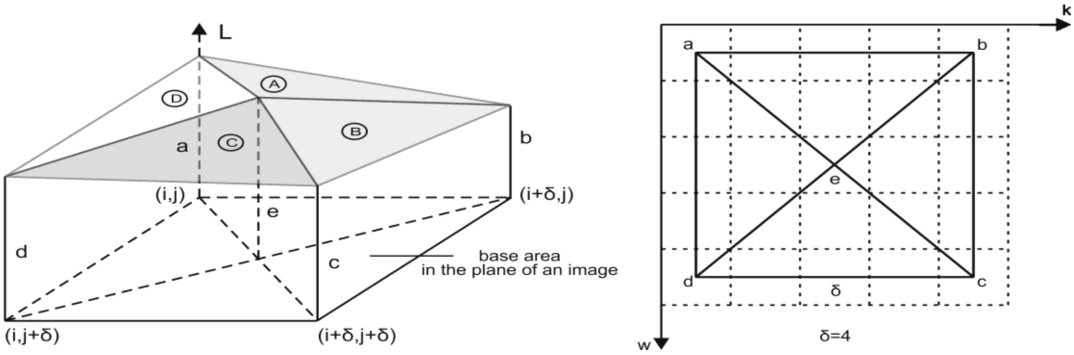

Fig. 2. The schema for estimating the component surface areas, covering the base area in the coordinate system: w – rows of the matrix, k – columns of the matrix, L – pixel value.

The base area is a square with the side δ, with length in pixels. The area may be divided with diagonals into 4 isosceles triangles on which the frustum prisms are then constructed. The lengths of their sides equal the value of pixels on which they stand (within the spectrum 0÷255). The prisms are covered by triangles (facets). The sum of the 4 areas of facets is a plane covering the base area.

The surface of the image is further divided into squares with sides of length δ= 3, 7, 9 and 25. For each square, the area of facets is calculated. The logarithms (base of 10) of the sum of the areas of a single-type windows create the set Y. The set X are logarithms (base of 10) of the reciprocals of the squares of side lengths δ. There exists a linear correlation between those variables. The calculated coefficient of linear regression (1) enables determining fractal dimension Df (2) for the entire image.

$$a = \frac{S_{xy}}{S_{xx}} \tag{1}$$

S_{xy} - covariance of *x* and *y* variables,, S_{xx} - variance of *x* variable.

$$D_f = a + 1 \tag{2}$$

Thus calculated dimension is burdened by the standard error of regression coefficient:

$$m_f = \sqrt{\frac{1}{n-2}\left(\frac{S_{yy}}{S_{xx}} - A^2\right)} \tag{3}$$

n – sample size.

The relevance of fractal dimension is dependent on verifying the hypothesis of independent variables, when regression coefficient equals 0. The calculated statistics:

$$t = \frac{|a|}{m_f} = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \tag{4}$$

r - Pearson correlation coefficient.

must be greater than the critical value. If the zero hypothesis is valid, statistics *t* has the distribution of *t*-Student with *n*-2 degrees of freedom. From the distribution table *t*-Student, it can be concluded, for the previously assumed relevance level α = 0.1, that the critical value is $t_{n-2,\alpha}$.

While dividing the image into smaller parts (windows), the algorithm for calculating local fractal dimension in movable windows was established.

Calculating of entropy

The second measure of texture applied for this work is entropy. The practical application of Haralick's method is complicated, as it requires generating the so-called matrix of events [8].

For the purpose of calculation, its simplified version was used. The entropy is defined by the formula:

$$E = -\sum_{i=1}^{n} p(i)\log_r p(i) \tag{5}$$

where:
$p(i)$ – probability of the occurrence of event i,
n – number of events in a given space,
r – basis of the logarithm (in the information theory, it usually equals 2).

The probability of value g pixel occurring in the window measuring $w\times k$ can be estimated in the following way:
The histogram of a digital image is a vector of n components. For the following g values, the value of component H_g is the number of pixels of the same value, which can be described by the formula below:

$$H_g = \sum_{w=1}^{n/k}\sum_{k=1}^{n/w} s(g,x_{w,k}) \quad \text{where: } s(g,x_{w,k}) = \begin{cases} 1 \; when \; x_{w,k} = g \\ 0 \; in\ the\ opposite\ case \end{cases} \tag{6}$$

The probability of value g pixel occurring is described by this formula:

$$p(g) = \frac{H_g}{w\times k} \tag{7}$$

Entropy can be interpreted as the uncertainty of a given elementary event occurring in the moment following. Its value was calculated in the same windows as Df was.

MODEL AREA

For the purpose of establishing model values of features of the defined classes of land cover, a few test areas were selected in the photograph. For each class, 3 squares of the size 125x125 pixels were chosen. Each area was then divided into smaller ones (25x25 pixels), and the value of fractal dimension and entropy (E) was calculated. The results were approximated by calculating the standard deviations (Table 1) and the matrix of variance and covariance in R,G,B space. On that basis, the Bhattacharyya distance and the measures of separability (Tables 2 and 3) were calculated, accordingly to the Jeffrey's-Matusita formula (JM)[9].

The JM measure changes in the spectrum of 0÷2 (it approaches value 2 asymptotically), and its interpretation is as follows: 0.00÷1.00 very bad, 1.00÷1.80 bad, 1.81÷1.90 good, 0.91÷2.00 very good separateness.

For each class, the ellipsoids of error were generated (Fig. 3), accordingly to Baran's algorithm. The components of the ellipsoids were calculated in such a way that the probability of pixels of the values within them was 50%. They served the purpose of illustrating the separability of models and establishing the method of classification.

Table 1. Fractal dimension and entropy of the model areas

	Fractal dimension					
	R		G		B	
	Fd	sFd	Fd	sFd	Fd	sFd
osiers	2.0936	0.0484	2.0925	0.0528	2.1035	0.0444
rushes	2.075	0.0395	2.0827	0.0419	2.0811	0.034
submergent plants	2.0437	0.01	2.0494	0.0154	2.0449	0.0094
bare basin floor	2.0318	0.017	2.0412	0.0114	2.0431	0.0122

	Entropy		
	R	G	B

Table 2. Bhattacharyya distance between the classes of land cover

	Fractal dimension				Entropy			
	osiers	rushes	submergent pla	bare basin floor	osiers	rushes	submergent pla	bare basin floor
osiers		0.9608	3.7276	3.5004		1.4611	4.0664	3.7221
rushes	0.1241		3.0888	2.8372	0.2921		2.6581	2.2994
submergent plants	1	0.7978		0.5688	1	0.6166		0.3832
bare basin floor	0.9281	0.7181	0		0.9068	0.5202	0	
	sum of the normalized distances=3.5681				sum of the normalized distances=3.3357			

The normalized distance values are located below the diagonals.

Table 3. Separability measures of the models of land cover categories

	Fractal dimension				Entropy			
	osiers	rushes	submergent plants	basin floor	osiers	rushes	submergent plants	bare basin floor
osiers		1.24	1.95	1.94		1.54	1.97	1.95
rushes			1.91	1.88			1.86	1.8
submergent plants				0.87				0.64
bare basin floor								

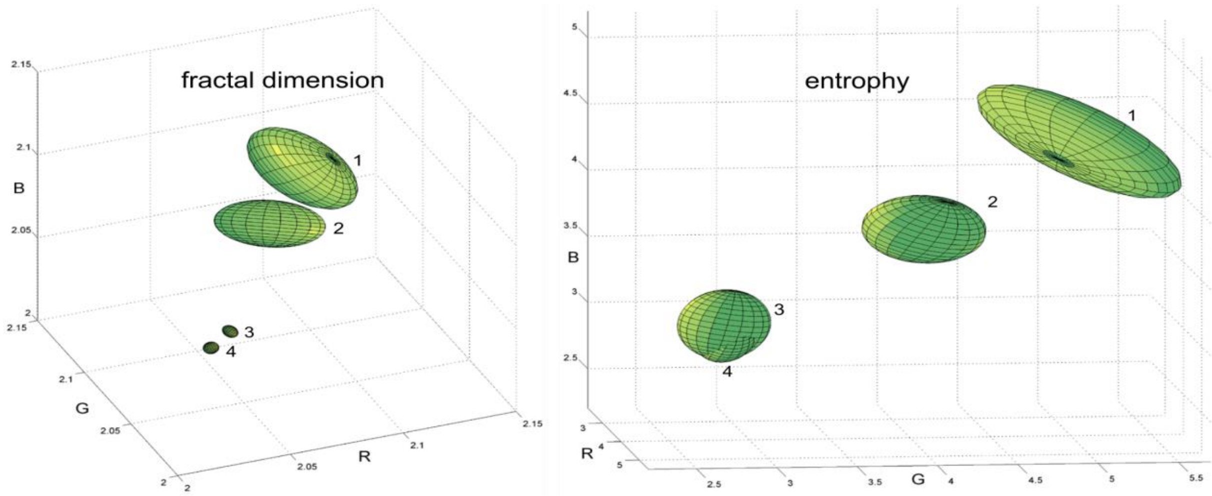

Fig. 3. Ellipsoids of error for the land cover classes. 1 – osiers, 2 – rushes, 3 – submerget vegetation, 4 – basin floor with no vegetation.

CLASSIFICATION AND OUTCOMES

The local values of Df and E were calculated in movable windows of 25x25 pixels. Thus created mosaics can bee seen in figure 4. Graphic depiction of the spatial structure of Df does not provide the researcher with any data substantial for visual interpretation of the clusters of vegetation. What is most visible are linear elements of landscape and defects in the shape of horizontal scores of emulsion. The groups or single birds are clearly marked by growth points of Df and entropy. The structure of entropy creates an image easier for interpretation of linear objects and transition zones between vegetation clusters.

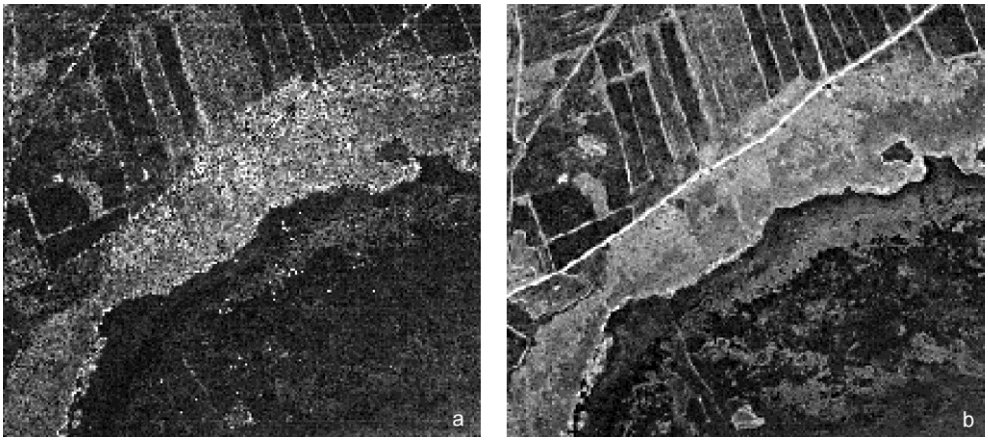

Fig. 4. Visualization of fractal dimension (a) and entropy (b) in red channel.

In the first stage, the only criterion for identification of a Df or E pixel with a given class was the presence of its value within the layer of a corresponding ellipsoid. Two images of the structure of land cover classes were thus generated. After visual analysis, the image created out of the Df structure was deemed insufficient. Far too many pixels were not assigned to any of the classes. The analysis of the image created out of the entropy structure (Fig. 4b) proved that substantial areas of meadows were classified along with submergent vegetation, and uncovered basin floor was classified as osiers and rushes.

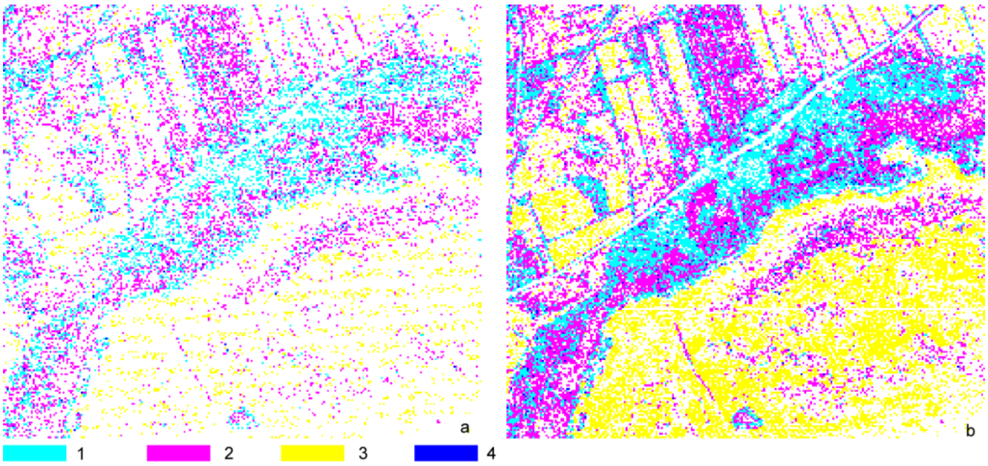

Fig. 4. The outcomes of classification: a – with the application of Df and E, b – with the application of E. 1 – osiers, 2 – rushes, 3 – submergent vegetation, 4 – pixels assigned to more than one class.

The assumed probability that a classified pixel would be identified with its proper class was 50% (Fig. 4a). With the assumption that a given pixel would be found within the proper ellipsoids of both those features, the probability of identification with the proper class was 75%. The outcome of such a classification is the image in Fig. 4a. The ultimate cost of increasing the credibility of classification is a higher number of unclassified pixels.

ANALYSIS OF RESULTS AND CONCLUSIONS

The traditional method of researching vegetation clusters is based upon differentiating the specific plant communities. A plant community, according to the phyto-sociologic rule of aquatic plants, is a patch (cluster) of dominant species on the area $\geq 1m^{2,}$, which covers that area in over 25% (≥3 in Braun-Blankuet's scale). The mapping occurs for given transects with the use of GPS data. That method relies on the experience and abilities of a researcher. Maps epitomize the subjective approach to the research and their analysis. The researched area is an extreme case which was encountered by the hydro-biologists cooperating with the author. Many scientific groups conducted their research there, and their endeavors resulted in maps varying to such an extent that it becomes impossible to find in them any indisputably credible data concerning the changes ongoing in the plant communities. The conducted research, whose aim was to establish the methodology of creating maps of plant communities' classification with the use of aerial photography and its analysis by means of fractal dimension and entropy, proved that the application of those two characteristics only is insufficient. The measures of model separability (Table 3) proved that submergent vegetation and bare basin floor are indistinguishable through the facet of those two features. This is illustrated in figure 3, where the ellipsoids of the classes marked 3 and 4 are very close to each other, or they overlap.

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